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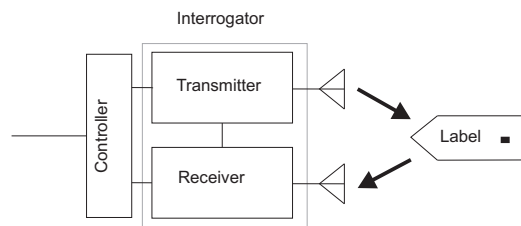
# Extending Coupling Volume Theory to Analyze Small Loop Antennas for UHF RFID Applications

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## INTRODUCTION

A simple illustration of the concept of a Radio Frequency Identification (RFID) system is provided in Fig. 1. Here a transmitter of interrogation signals which is contained within an interrogator communicates via electromagnetic waves with an electronically coded label to elicit from the label a reply signal containing useful data characteristic of the object to which the label is attached. The reply signal is detected by a receiver in the interrogator and made available to a control system.



**Fig. 1 Illustration of an RFID system**

There is a wide range of operating principles for such a system [1]. The operating principle and operating frequency are driven principally by the application of the labelling system; the constraints provided by electromagnetic compatibility regulations, environmental noise, and the ability of fields to permeate a scanned region of space or to penetrate intervening materials. Applications are found in reliable and secure data collection, object or personal identification, authentication, anti-counterfeiting, theft detection and the detection of location of the scanned objects.

In a primary category of passive systems the most common operating principle is that of RF backscatter [1] in which a powering signal or communication carrier supplies power or command signals via an HF or UHF link. However the circuits within the label operate at RF or lower, and reply via sidebands generated by modulation, within the label, or part of the powering carrier. This approach combines the benefits of relatively good propagation of signals at HF and UHF and the low power of operation of microcircuits at RF or lower. Powering at UHF is employed when a longer interrogation range (several meters) is required, and HF powering is employed when electromagnetic fields, which exhibit good material penetration and sharp spatial field confinement, are required or sometimes when a very low cost RFID system implementation is desired.

A large portion, approximately 3 – 4 cents, of the label cost is allocated to the antenna manufacture, antenna and IC assembly and packaging in the manufacture of RFID labels. Hence there is a keen interest to produce small antennas to reduce manufacturing costs. This paper presents novel method of analysing the effectiveness of small antennas for UHF RFID labels using coupling volume theory.

The subject of small antennas has been considered in the past in notable publications [2, 3, 4 and 5]. This analysis makes use of both coupling volume theory and radiating antenna theory to analyse small antennas for UHF RFID use. Radiating antenna theory, commonly used in radar calculations, is appropriate in the context in which labels are placed in the far field, and when the label antenna size is large enough for the theoretically available source power from a lossless antenna to be actually extracted, or nearly so, within the constraints imposed by the facts that there is some loss, and complete extraction of the available source power is not possible and that the radiation resistance of the antenna is accompanied by a reactance which results in the fact that good transfer of power to an external load can only be accomplished over a limited bandwidth.

Coupling volume theory, first published in [6], is a powerful dimensionless analysis tool with a number of applications presented in [7 and 8] and the theory was devised for situations in which labels are placed in the near field, i.e. the energy storage field of a transmitter antenna, and also in the situations in which the radiation resistance of the label

antenna is small in relation to the losses in that antenna. For operation in the HF ISM band centred at 13.56 MHz, both of these conditions are normally satisfied. For the situation when labels are placed in the far field of an interrogator antenna, but the labels are so small that their own losses are large in relation to the radiation resistance of the label antenna, it is appropriate to use a hybrid version of radiating antenna theory and coupling volume theory. Radiating antenna theory is used to calculate the energy density at the label position, and coupling volume theory is used to work out what useful power the label antenna can extract from the field.

## MEASURES OF EXCITING FIELD

In the analysis of the performance of RFID systems it is important to consider whether the labels are placed in the near field (energy storage) or far field (energy propagating) fields of the interrogator antenna. When the antenna is of small gain the boundary between the near and far field is expressed in the more familiar terms of radian sphere of radius  $r = \lambda/(2\pi)$ , where  $\lambda$  is the frees space electromagnetic wavelength at the operating frequency.

It is possible to develop the two measures of field excitation for linearly polarised magnetic field described by a real r.m.s phasor  $\mathbf{H}$  as given in (1) and (2).

$$\text{Radial component of Poynting vector } S_r = \eta |\mathbf{H}|^2 \text{ Wm}^{-2} \quad (1)$$

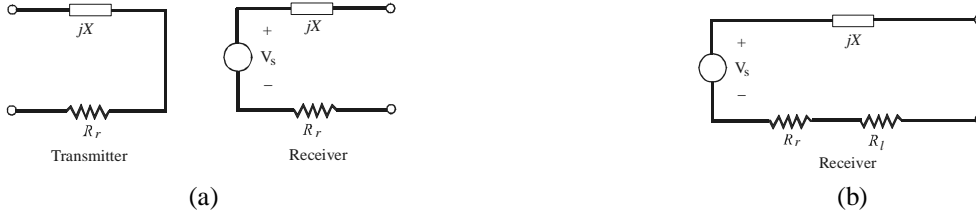
$$\text{Volume density of reactive power } W_v = \omega \mu_0 |\mathbf{H}| \text{ VAm}^{-3} \quad (2)$$

Equation (2) is  $\omega$  times the peak value of stored magnetic energy per unit volume. Hence if  $\beta$  is the propagation constant at the frequency under consideration then volume density reactive power can be expressed as in (3) using  $S_r$  which is the radial component of Poynting vector giving the power like quantity for far field radiation.

$$W_v = \beta S_r \quad (3)$$

Equation (3) is a far field expression and there is no near field reactive energy storage field to augment  $W_v$ .

## SMALL ANTENNAS FOR RFID APPLICATIONS



**Fig. 2 RFID label antenna equivalent with a (a) lossless antenna and (b) accounting for ohmic losses.**

Fig. 2(a) shows equivalent circuits for an electrically small antenna, operating at the left in its transmitting role, and at the right in its receiving role. It shows that in both cases there is a radiation resistance in series with an antenna reactance. The same resistance and reactance is found in both circuits. It has been shown that for ideal lossless and electrically small antennas, which would be enclosed completely by a sphere of radius  $r$ , (both electric and magnetic dipole antennas), the radiation quality factor scales as in (4) [3].

$$Q_r = (\beta r)^{-3} + (\beta r)^{-1} \quad (4)$$

$$R_r = 20\pi^2 (\beta a)^4 \Omega \quad (5)$$

While the  $Q_r$  of a practical antenna will be less than the minimum bound in (4), due to material losses, (4) does show that as the antenna becomes very small, a relatively large and increasing reactance stands between the radiation resistance and any external load to which we might wish to match. When losses are to be taken into account the antenna will also have a loss resistance  $R_l$  so its equivalent circuit becomes modified to that shown in Fig. 2(b). The optimum load impedance, previously  $R_r - jX$ , now becomes  $R_r + R_l - jX$ , and the power which can be delivered to that load impedance is reduced. When the antenna becomes very small, the radiation resistance of a loop antenna of radius  $a$  given by (5) reduces, however  $R_r \ll R_l$ . It is in this situation we have the option of applying coupling volume theory to determine the

circuit behaviour. In coupling volume theory, the source voltage in the above antenna circuit might as well be calculated from Faraday's law, the radiation resistance neglected, the self inductance calculated from the magnetostatic formula, and the loss resistance be determined taking into account that conduction will only occur within a skin depth of the metal surface.

## FAR FIELD RELATIONS

For calculation of the power  $P_r$  coupled in the far field to a label with a lossless receiving antenna the usual approach is to derive the available source power from the label antenna from (6) where  $A_e$  and  $g_r$  are the effective area of the label and gain of the label antenna, while  $g_t$  is the label antenna gain.

$$P_r = S_r A_{er} = \frac{g_r \lambda^2}{4\pi} S_r \quad (6)$$

The effective area for the far field is a concept unrelated to either a magnetic flux collection area or a electric flux collection area. It is unrelated to any physical area the antenna may posses, but has it has the desirable property that it is possible to imagine that the label antenna collects all of the radiated power which flows through that effective area which may be thought of as surrounding the label antenna.

$$S_r = \frac{g_t P_t}{4\pi r^2} = \eta |H|^2 \text{ Wm}^{-2}. \quad (7)$$

Equation (7), where  $P_t$  is the power transmitted and  $r$  is the distance from the transmitter antenna to the label position assuming that the label has been placed in the direction of strongest radiation from the RFID interrogator (transmitter) antenna defines the power flow per unit area defined by the radial component of the Poynting vector. Then the Lorenz reciprocity theorem of electrodynamic may be used to show that the effective area of a receiving antenna is related to the gain  $g_r$  it would have in a transmitting role by the equations

$$A_{er} = \frac{g_r \lambda^2}{4\pi} \quad (8)$$

$$\frac{P_r}{P_t} = g_r g_t \left( \frac{\lambda}{4\pi r} \right)^2 = \frac{A_{er} A_{et}}{\lambda^2 r^2} \quad (9)$$

Equation (9) provides the result usually used to evaluate the power extracted by an RFID label in the far field when  $R_r \gg R_l$  for a label antenna.

## ANALYSIS OF A SMALL LOOP

Considering a small loop antenna with a radiation resistance  $R_r$  and self inductance  $L$  with an ohmic resistance  $R_l$ , gain  $g_d$ , excited by the magnetic field of an interrogator antennas has an effective area  $A_e$  of

$$A_e = g_d \frac{\lambda^2}{4\pi}. \quad (10)$$

and neglecting losses, the available source power  $P_a$  when the loop is in a field of Poynting vector  $S_r$  is given in (11). This is the power which a lossless loop antenna would deliver to a load  $R_L = R_r$  and is identical to that evaluated in (9).

$$P_a = \frac{g_d \lambda^2}{4\pi} S_r \quad (11)$$

However with electrically small antennas where the antenna is small in relation to a wave length, it has been shown [9] that  $R_r \ll R_l$ . Hence it is possible to focus attention on  $R_l$  neglecting  $R_r$  completely. Thus a more useful and meaningful formulation of the power available from such an antenna may be expressed using the coupling volume theory.

The coupling volume  $V_c$  of a label antenna can be defined as in (12), while the coupling volume for a planar coil of  $N$  turns area  $A$  and self inductance  $L$  is given by (13)[6].

$$V_c = \frac{\text{Reactive power in the label inductor when short circuit}}{\text{Reactive power density per unit volume at label position}} \quad (12)$$

$$V_c = \frac{\mu_0 N^2 A^2}{L}. \quad (13)$$

Using coupling volume theory, which can be apply when  $R_r$  is negligible with respect to the losses  $R_l$ , it is appropriate, in view of the definition in (12), to calculate the power delivered to the losses  $R_l$  without any external load yet having been added. This power  $P_c$  is given by

$$P_c = \left( \frac{4R_r}{R_l} \right) P_a \quad (14)$$

Substitute for  $R_r$ ,  $P_a$  and  $S_r$  from (5), (11) and (7) respectively and using the value 1.5 for  $g_d$ , provides

$$P_c = \frac{\eta^2 \beta^2 |H|^2 A^2}{2R_l}. \quad (15)$$

In order to manipulate this into a more familiar form, replace  $R_l$  by  $\omega L/Q$  where  $Q$  is the quality factor of the antenna inductor and  $\eta\beta$  by  $\omega\mu_0$  and obtain

$$P_c = \frac{(\omega\mu_0 |H|^2)}{2} \left( \frac{\mu_0 A^2}{L} \right) Q. \quad (16)$$

Equation (16) can be expressed using (3) and (13) into the more familiar form in (17).

$$P_c = QW_v V_c \quad (17)$$

Equation (17) is the standard form of the result from coupling volume theory for coils coupling to the magnetic field [6]. Thus the effective area and coupling volume formulations of loop antenna behaviour are entirely equivalent. However the coupling volume theory formulation emphasises the internal antenna losses of the label antenna to provide a more meaningful result for the power available from a small loop antenna in the far field.

#### AVAILABLE POWER FROM A SMALL LOOP

When an optimum load resistor  $R_L = R_c$  is added to a small loop, the power which can be delivered to that load is one quarter of that given by (15), if  $Q$  is interpreted as defined by the loop losses and its inductance. If however redefining  $Q$  to be the new and lowered  $Q$ , determined by the sum of the loop losses and the damping of the external load, then the power that can be delivered to a matched load is half that given by (15).

#### CONCLUSIONS

The Poynting vector-effective area formulation and the coupling volume formulations are apparently dissimilar, however they are both equivalent and useful in different contexts. The difference between the formulations is whether they emphasise the radiation resistance or the internal losses of the label antenna.

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